

B -Meson Distribution Amplitude from the $B \rightarrow \pi$ Form Factor

Alexander Khodjamirian, Thomas Mannel and Nils Offen

*Theoretische Physik 1, Fachbereich Physik, Universität Siegen,
D-57068 Siegen, Germany*

Abstract

Employing the light-cone sum rule approach in QCD, we relate the B -meson distribution amplitude to the $B \rightarrow \pi$ form factor at zero momentum transfer. In leading order, the sum rule is converted into a simple expression for the inverse moment λ_B of the distribution amplitude ϕ_+^B . Using as an input the $B \rightarrow \pi$ form factor calculated from the light-cone sum rule in terms of pion distribution amplitudes, we obtain an estimate: $\lambda_B = 460 \pm 160$ MeV. We investigate how this result is modified by the B -meson three-particle distribution amplitudes.

1 Introduction

The B -meson distribution amplitudes (DA's) emerge as universal nonperturbative objects in many studies of exclusive B meson decays [1, 2, 3], in particular, in $B \rightarrow \gamma l \nu_l$ [4]. Still there is a very limited knowledge of the nonperturbative parameters determining these DA's. Especially important is the inverse moment of the two-particle DA ϕ_+^B (see the definition below) which enters many factorization formulas, including QCD factorization for hadronic [5] and radiative [6] B decays.

As usual, when there is a need to calculate some hadronic parameter, the method of QCD sum rules [7] turns out to be helpful. The “classical” two-point sum rule, based on the local OPE and condensate expansion, was used for the B -meson DA's already in the original study [2]. Recently, the two-point sum rule calculation was improved in [8], including next-to-leading order corrections. An estimate of the inverse moment of ϕ_+^B was also obtained in [9], matching the factorization formula to the light-cone sum rule for $B \rightarrow \gamma l \nu_l$. Currently, the spread of model assumptions and sum rule predictions for the B meson DA's in the literature remains rather large. An additional QCD estimate, even an approximate one, can be useful in that respect.

In this paper we suggest a new approach, using the light-cone sum rule technique [10] and relating the B -meson DA to the $B \rightarrow \pi$ form factor. The latter is a better known hadronic object, receiving a lot of attention in recent years. At small momentum transfers this form factor was calculated in QCD using light-cone sum rules [11], based on OPE in terms of the pion DA's with growing twist. Therefore, the link between the B -meson DA and $B \rightarrow \pi$ form factor established below, provides an independent dynamical information on the B -meson DA.

The aim of this letter is mainly to outline the procedure and to demonstrate that it works at the leading-order level. Gluon effects originating from the three-particle Fock states of B meson are also investigated at a qualitative level, within our very limited knowledge of the quark-antiquark-gluon DA's. Perturbative corrections remain beyond the scope of this work, being postponed to a future study. Hence we also do not take into account the nontrivial renormalization of the B -meson DA worked out in [12] (see also [8]).

2 Correlation function

The starting object in our approach is the correlation function which represents a product of the heavy-light and light quark currents sandwiched between the vacuum and the B -meson states:

$$F_{\mu\nu}^{(B)}(p, q) = i \int d^4x \, e^{ip \cdot x} \langle 0 | T \{ \bar{d}(x) \gamma_\mu \gamma_5 u(x), \bar{u}(0) \gamma_\nu b(0) \} | \bar{B}^0(p+q) \rangle. \quad (1)$$

This expression resembles the correlation function used in deriving the light-cone sum rules for $B \rightarrow \pi$ form factor [11], with the roles of B and π interchanged: in (1) the B meson is on shell, $(p+q)^2 = m_B^2$, and the pion is interpolated by the axial-vector current. If the

momentum squared of the axial-vector current is spacelike and sufficiently large:

$$p^2 < 0, \quad |p^2| \sim 1 \text{ GeV}^2 \gg \Lambda_{QCD}^2, \quad (2)$$

the intermediate u quark is highly virtual and propagates near the light-cone $x^2 = 0$. The momentum transfer squared in the heavy-light vertex will be put to zero ($q^2 = 0$). In this kinematical configuration, the product of currents in the correlation function (1) can be expanded near the light-cone. Note that, up to the differences in the quantum numbers of currents, the correlation function (1) is similar to the $B \rightarrow \gamma l \nu_l$ amplitude. For the latter, the light-cone dominance is usually assumed (in the framework of the heavy-quark mass expansion) [4] at the small mass squared q^2 of the lepton pair, (that is, at large photon energy in the B -meson rest frame). However, the long-distance effects for the real photon ($p^2 = 0$), although $1/m_b$ suppressed, are still present ¹. With our choice (2) these effects in the correlation function (1) are additionally suppressed by inverse powers of $|p^2|$.

In the leading order, only one tree-level diagram contributes (Fig. 1a), where the virtual u quark is replaced by the free propagator:

$$\langle 0 | T \{ u_\alpha(x) \bar{u}_\beta(0) \} | 0 \rangle = \frac{i \not{x}_{\alpha\beta}}{2\pi^2(x^2)^2}. \quad (3)$$

Contracting the u -quark fields in (1), we encounter the vacuum- B -meson matrix element of a bilocal operator, with the light-antiquark and heavy-quark fields at a near-to-light-cone separation. This hadronic matrix element is decomposed in the B -meson two-particle DA's [2, 3] we are interested in :

$$\begin{aligned} & \langle 0 | \bar{d}_\alpha(x) [x, 0] b_\beta(0) | \bar{B}^0(v) \rangle \\ &= -\frac{if_B m_B}{4} \int_0^\infty d\omega e^{-i\omega v \cdot x} \left[(1 + \not{v}) \left\{ \phi_+^B(\omega) - \frac{\phi_+^B(\omega) - \phi_-^B(\omega)}{2v \cdot x} \not{x} \right\} \gamma_5 \right]_{\beta\alpha}, \end{aligned} \quad (4)$$

where v is the B -meson velocity ($p + q = m_B v$), $[x, 0]$ is the path-ordered gauge factor, and ϕ_\pm^B are the distribution amplitudes in the momentum space. The variable $\omega > 0$ is the plus component of the momentum of the light spectator quark in the B meson. Note that the DA's are essentially concentrated around $\omega \sim \bar{\Lambda}$, where $\bar{\Lambda} = m_B - m_b$.

Neglecting the three-particle Fock states of B meson, one obtains [3, 14] a simple differential relation of the Wandzura-Wilczek (WW) type between the two functions ϕ_+^B and ϕ_-^B , which can also be written in the following form:

$$\phi_-^B(\omega) = \int_\omega^\infty d\rho \frac{\phi_+^B(\rho)}{\rho}. \quad (5)$$

Using the definition (4), we easily derive the leading-order answer for the light-cone expansion of the correlation function (1). In what follows, we are interested in the invariant

¹In the LCSR approach to $B \rightarrow \gamma l \nu_l$ [9, 13] the long-distance photon emission is described by the photon DA and at finite m_b turns out to be large.

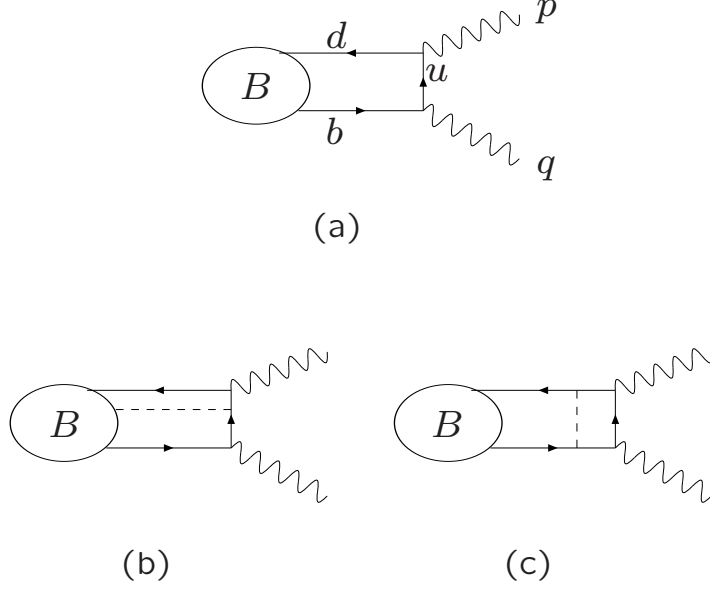


Figure 1: *Diagrams corresponding to the correlation function (1): (a) the leading-order diagram, (b) the soft-gluon emission with 3-particle DA's, (c) one of the diagrams with the hard gluon exchange. The wavy (dashed) lines denote external currents (gluons).*

amplitude multiplying $p_\mu p_\nu$. After rearranging the part of the DA containing $v \cdot x$ in the denominator by means of partial integration, the answer for this amplitude turns out to be surprisingly simple, in particular, the parts proportional to ϕ_+^B cancel each other. The result reads:

$$F_{\mu\nu}^{(B)}(p, q) = 2if_B \int_0^\infty \frac{d\omega}{m_B\omega - p^2} \phi_-^B(\omega) p_\mu p_\nu + \dots, \quad (6)$$

where ellipses denote the remaining Lorentz structures.

3 Sum rule for the inverse moment

The OPE result (6) for the correlation function (1) will now be used to derive LCSR for $B \rightarrow \pi$ form factor. The procedure is similar to the treatment of the vacuum-pion correlation function in [11]. The roles of the pion and B meson are now reversed: the pion is generated by the interpolating current and has to be approximated by quark-hadron duality, whereas the on-shell B meson is represented by its DA.

We write down a hadronic dispersion relation in the channel of the axial-vector current with the momentum squared p^2 . The set of hadronic states contributing to that relation includes the pion, a_1 meson, excited resonances and continuum states with $J^{PC} = 0^{-+}, 1^{++}$.

Isolating the ground-state pion contribution from the rest, we obtain, at $q^2 = 0$:

$$F_{\mu\nu}^{(B)}(p, q) = \left\{ \frac{2if_\pi f_{B\pi}^+(0)}{-p^2} + \int_{s_h}^{\infty} ds \frac{\rho^h(s)}{s - p^2} \right\} p_\mu p_\nu + \dots, \quad (7)$$

where the standard definitions of the pion decay constant $\langle 0 | \bar{d} \gamma_\mu \gamma_5 u | \pi(p) \rangle = ip_\mu f_\pi$ and $B \rightarrow \pi$ form factors

$$\langle \pi(p) | \bar{u} \gamma_\nu b | B(p+q) \rangle = (2p+q)_\nu f_{B\pi}^+(q^2) + q_\nu f_{B\pi}^-(q^2) \quad (8)$$

are used. Furthermore, in (7) we neglect the pion mass and denote by ρ^h the spectral density of all states heavier than the pion, with the corresponding hadronic threshold s_h . In the channel of the axial-vector current quark-hadron duality works reasonably well, as known already from the original paper [7] on QCD sum rules. In particular, f_π is reproduced if one attributes to the pion the integral over the spectral density calculated from OPE with the threshold $s_0^\pi = 0.7 \text{ GeV}^2$ (see also the review [15]). The same duality ansatz was used in LCSR for the pion e.m. form factor [16, 17].

Note that the expression (6) for the amplitude $F_{\mu\nu}^B$ is easily transformed into a form of dispersion relation, if one replaces the variable ω by $s = m_B \omega$:

$$F_{\mu\nu}^{(B)} = \left\{ \int_0^\infty \frac{ds}{s - p^2} \rho^{OPE}(s) \right\} p_\mu p_\nu + \dots, \quad \text{with} \quad \rho^{OPE}(s) = \frac{2if_B}{m_B} \phi_-^B(s/m_B). \quad (9)$$

Equating the above result to the hadronic dispersion relation (7) at large spacelike p^2 , we employ the quark-hadron duality approximation: $\rho^h(s) \Theta(s - s_h) = \rho^{OPE}(s) \Theta(s - s_0^\pi)$ and apply the Borel transformation to both parts of this equation. This transformation replaces $|p^2|$ with the effective scale $M^2 \sim 1 \text{ GeV}^2$. Simultaneously, the Borel exponent suppresses the contribution of heavier than pion hadronic states in the dispersion integral, so that the resulting sum rule is less sensitive to the accuracy of the duality approximation. Finally, we obtain:

$$\int_0^{s_0^\pi} ds e^{-s/M^2} \phi_-(s/m_B) \simeq \phi_-(0) \int_0^{s_0^\pi} ds e^{-s/M^2} = \frac{f_\pi f_{B\pi}^+(0)}{f_B} m_B, \quad (10)$$

where we used the fact that $\sqrt{s_0^\pi} \sim \bar{\Lambda}$ and $s_\pi^0 \ll m_B^2$.

This relation has a very transparent physical interpretation: in $B \rightarrow \pi$ transition the pion (in the absence of hard gluon exchange) is produced via end-point mechanism, that is, picking up the light spectator-quark of B meson with a vanishing momentum fraction.

Using the relation between $\phi_-(0)$ and ϕ_+ in the form (5) we encounter on l.h.s. of (10) the inverse moment defined as

$$\int_0^\infty d\omega \frac{\phi_+(\omega)}{\omega} = \frac{1}{\lambda_B}. \quad (11)$$

Hence, an approximate relation for λ_B can be established:

$$\frac{1}{\lambda_B} = \frac{f_\pi f_{B\pi}^+(0) m_B}{f_B M^2 (1 - e^{-s_0^\pi/M^2})}, \quad (12)$$

which is valid up to unaccounted $O(\bar{\Lambda}/m_B) \sim O(\sqrt{s_0^\pi}/m_B)$ corrections. The normalization scale μ of λ_B in this relation can only be roughly estimated. Since the average virtuality of the intermediate u -quark in the correlator is between M^2 and $\bar{\Lambda}m_b$, we expect that $1 \text{ GeV} < \mu < \sqrt{\bar{\Lambda}m_B}$.

4 Numerical estimate of λ_B

To obtain a numerical estimate of λ_B from (12), we take as an input the threshold parameter $s_0^\pi = 0.7 \text{ GeV}^2$ and the interval $M^2 = 0.5 \div 1.2 \text{ GeV}^2$, used in the QCD sum rule for the pion channel [7, 15].

Concerning $B \rightarrow \pi$ form factor and f_B , it is natural to take the estimates obtained from LCSR (in terms of pion DA's) and from the two-point QCD sum rule, respectively. Strictly speaking, since the radiative corrections are not yet included, the quantity $f_{B\pi}^+(0)$ in (12) has to be interpreted as the “soft” (end-point or nonfactorizable) part of the form factor. On the other hand, the LCSR analysis [11] predicts that at finite m_b the “hard-scattering” (factorizable) part of the $B \rightarrow \pi$ form factor determined by the radiative corrections to the sum rule, is subdominant (see also [18]). Therefore, we take the complete form factor predicted from LCSR. More specifically, to obtain the ratio $f_{B\pi}^+(0)/f_B$ entering (12), we calculate the product $f_{B\pi}^+(0)f_B$ from LCSR and divide it by the two-point sum rule for f_B^2 , both taken with $O(\alpha_s)$ accuracy. This calculation uses the same input as in [19] (see also [20]) and, in particular, yields $f_{B\pi}^+(0) = 0.26 \pm 0.06$ ². The remaining parameters in (12) are $m_B = 5.279 \text{ GeV}$ and $f_\pi = 131 \text{ MeV}$. Our final result is

$$\lambda_B = 460 \pm 160 \text{ MeV}, \quad (13)$$

where the uncertainties due to the variation of input parameters are added in quadrature. Including gluonic corrections to (12) will allow one to improve the above estimate. However the achievable accuracy of the $B \rightarrow \pi$ form factor and f_B puts a lower limit of about $\pm 20\%$ on the uncertainty in this relation.

Our prediction reveals an encouraging agreement with the result of the 2-point sum rule calculation [8] $\lambda_B = 460 \pm 110 \text{ MeV}$, being also consistent with $\lambda_B = 350 \pm 150 \text{ MeV}$ adopted in QCD factorization approach [5] and with the estimate $\lambda_B \simeq 600 \text{ MeV}$ inferred [9] from a LCSR for $B \rightarrow \gamma l \nu_l$.

5 Gluon Corrections

The three-particle (quark-antiquark-gluon) Fock states of B meson influence the relation (12) in two ways: 1) directly, via the gluon emission from the virtual quark line (diagram

²This interval agrees with the recent LCSR estimate $f_{B\pi}^+(0) = 0.258 \pm 0.031$ [21].

in Fig. 1b), and 2) indirectly, due to modification of the WW relation (5). Since very little is known about the B -meson three-particle DA's, our analysis of both effects presented in this section is rather qualitative.

We calculated the diagram in Fig. 1b, inserting in the correlator (1) the one-gluon part of the u -quark propagator near the light-cone [22]. Instead of Eq. (3) one then uses

$$\langle 0 | T\{u_\alpha(x)\bar{u}_\beta(0)\} | 0 \rangle = -\frac{i}{16\pi^2 x^2} \int_0^1 du G_{\tau\rho}(ux) (\not{x}\sigma^{\tau\rho} - 4iu x_\tau \gamma_\rho)_{\alpha\beta}, \quad (14)$$

where $G_{\tau\rho} = g_s G_{\tau\rho}^a (\lambda^a/2)$ and the Fock-Schwinger gauge is adopted, with the path-ordered gauge factor equal to the unit. The vacuum-to- B matrix element is then decomposed in four independent three-particle DA's introduced in [14] in terms of the most general parameterization compatible with the heavy-quark limit:

$$\begin{aligned} \langle 0 | \bar{d}_\alpha(x) G_{\lambda\rho}(ux) b_\beta(0) | \bar{B}^0(v) \rangle &= \frac{f_B m_B}{4} \int_0^\infty d\omega \int_0^\infty d\xi e^{-i(\omega+u\xi)v \cdot x} \\ &\times \left[(1 + \not{v}) \left\{ (v_\lambda \gamma_\rho - v_\rho \gamma_\lambda) \left(\Psi_A(\omega, \xi) - \Psi_V(\omega, \xi) \right) - i\sigma_{\lambda\rho} \Psi_V(\omega, \xi) \right. \right. \\ &\left. \left. - \left(\frac{x_\lambda v_\rho - x_\rho v_\lambda}{v \cdot x} \right) X_A(\omega, \xi) + \left(\frac{x_\lambda \gamma_\rho - x_\rho \gamma_\lambda}{v \cdot x} \right) Y_A(\omega, \xi) \right\} \right]_{\beta\alpha}. \end{aligned} \quad (15)$$

In the above, Ψ_V, Ψ_A, X_A and Y_A are the DA's in the momentum space, the variables $\omega > 0$ and $\xi > 0$ being, respectively, the plus components of the light-quark and gluon momenta in the B meson.

In the following, we only take into account the first two DA's, Ψ_V and Ψ_A . The contribution of the remaining two DA's to the sum rule is suppressed, at least by the inverse power of the Borel parameter. The result of our calculation yields the following addition to the correlation function (6):

$$\begin{aligned} F_{\mu\nu}^{(B)\{\Psi_V, A\}}(p, q) &= 2if_B \int_0^\infty d\omega \int_0^\infty d\xi \int_0^1 du \frac{m_B}{(m_B - \omega - u\xi)(m_B(\omega + u\xi) - p^2)^2} \\ &\quad \left(\Psi_V(\omega, \xi) + (1 - 2u)\Psi_A(\omega, \xi) \right) p_\mu p_\nu + \dots, \end{aligned} \quad (16)$$

We assume that both Ψ_V and Ψ_A vanish at large ω, ξ , so that all integrals are ultra-violet convergent³. Furthermore, employing partial integration, we transform the above

³Since we work in LO approximation, the effect of “radiative tail” [2, 12, 8] originating from the hard gluon emission is neglected here.

expression to the dispersion form and apply duality approximation as well as Borel transformation. The sum rule (10) becomes:

$$\int_0^{s_0^\pi} ds e^{-s/M^2} \left\{ \phi_-(s/m_B) + \frac{d}{ds} \left[\frac{m_B^2}{m_B^2 - s} \int_0^{s/m_B} d\omega \int_{s/m_B - \omega}^\infty \frac{d\xi}{\xi} \left(\Psi_V(\omega, \xi) + \left[1 - 2 \left(\frac{s - m_B \omega}{\xi m_B} \right) \right] \Psi_A(\omega, \xi) \right) \right] \right\} = \frac{f_\pi f_{B\pi}^+(0)}{f_B} m_B. \quad (17)$$

To assess the relative role of the gluon correction, it is sufficient to establish the behavior of both DA's, Ψ_V and Ψ_A , at small ω and ξ . We follow the same approach as in [2] and compare the decomposition (15) valid in the heavy quark limit with the well known definitions of quark-antiquark-gluon DA's $\varphi_{3P}(\alpha_i)$, $\varphi_\perp(\alpha_i)$, $\varphi_\parallel(\alpha_i)$, $\tilde{\varphi}_\perp(\alpha_i)$, and $\tilde{\varphi}_\parallel(\alpha_i)$, of a pseudoscalar meson \mathcal{P} with finite mass, where $\alpha_i \equiv \alpha_1, \alpha_2, \alpha_3$ and α_1, α_3 are the fractions of the meson momentum carried by the light quark and gluon, respectively. For example φ_{3P} is defined as

$$\begin{aligned} \langle 0 | \bar{d}(x) G_{\lambda\rho}(ux) \sigma_{\mu\nu} b(0) | \mathcal{P}(P) \rangle &= i f_{3P} \left[\left(P_\lambda P_\mu g_{\rho\nu} - P_\lambda P_\nu g_{\rho\mu} \right) - \{ \rho \leftrightarrow \lambda \} \right] \\ &\times \int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_3 e^{-i(\alpha_1 + u\alpha_3)P \cdot x} \varphi_{3P}(\alpha_i) |_{\alpha_2=1-\alpha_1-\alpha_3}. \end{aligned} \quad (18)$$

where f_{3P} is the nonperturbative normalization parameter, P is the meson momentum and DA has the following asymptotic form:

$$\varphi_{3P}(\alpha_i) \sim \alpha_1 \alpha_2 \alpha_3^2. \quad (19)$$

For the sake of brevity we do not quote here definitions and asymptotic forms of all other DA's, they can be found, e.g. in the Appendix B of [17]. It is important that 3-particle DA's are proportional at least to the first power of the gluon momentum fraction α_3 . From that we expect that, at $\xi \rightarrow 0$, also $\Psi_{A,V}(\xi) \sim \xi^n$ with $n \geq 1$. The latter condition is sufficient for the infrared convergence of the integrals over ω, ξ in (17). Furthermore, we find that the gluon correction in (17) is at least $\sim 1/m_B$ suppressed with respect to the leading-order term proportional to ϕ_- .

Our final comment concerns the violation of the WW relation which, in the presence of three-particle DA's, acquires a correction worked out in [14]. In particular the relation for the inverse moment modifies to:

$$\phi_-(0) = \frac{1}{\lambda_B} + \int_0^\infty \frac{d\omega}{\omega} I(\omega) \quad (20)$$

where

$$I(\omega) = 2 \frac{d}{d\omega} \int_0^\omega d\rho \int_{\omega-\rho}^\infty \frac{d\xi}{\xi} \frac{\partial}{\partial \xi} \left(\Psi_V(\rho, \xi) - \Psi_A(\rho, \xi) \right) \quad (21)$$

To deduce the low-energy behavior of the DA's in this integral one may again rely on comparison with the 3-particle DA's at finite mass. Multiplying (15) by $\sigma_{\mu\nu}\gamma_5$ and taking trace, we found that the resulting kinematical structure coincides with the one in (18). Hence, we assume that the behavior at small $\omega/m_B \rightarrow 0$ and $\xi/m_B \rightarrow 0$ of $(\Psi_V(\omega, \xi) - \Psi_A(\omega, \xi))$ is the same as the behavior of $\varphi_{3P}(\alpha_i)$ at $\alpha_1 \rightarrow 0$ and $\alpha_3 \rightarrow 0$, respectively, yielding, according to (19):

$$\Psi_V(\omega, \xi) - \Psi_A(\omega, \xi) \sim \varphi_{3\pi}(\alpha_1, 1 - \alpha_1 - \alpha_3, \alpha_3)|_{\alpha_1=\omega/m_B, \alpha_3=\xi/m_B} \sim \omega\xi^2, \quad (22)$$

so that the integral in (20) is convergent. In order to estimate its numerical value we use the normalization factors

$$\begin{aligned} \int_0^\infty d\omega \int_0^\infty d\xi \Psi_V(\omega, \xi) &= \frac{\lambda_H^2}{3} = (0.18 \pm 0.07)/3 \text{ GeV}^2, \\ \int_0^\infty d\omega \int_0^\infty d\xi \Psi_A(\omega, \xi) &= \frac{\lambda_E^2}{3} = (0.11 \pm 0.06)/3 \text{ GeV}^2, \end{aligned} \quad (23)$$

estimated in [2] by relating them to the matrix elements of certain local operators and calculating these matrix elements from the sum rules in HQET.

We adopt a simple model for the difference of the two DA's:

$$\Psi_V(\omega, \xi) - \Psi_A(\omega, \xi) = \frac{(\lambda_H^2 - \lambda_E^2)}{6\bar{\Lambda}^5} \omega \xi^2 \exp\left(-\frac{\omega + \xi}{\bar{\Lambda}}\right), \quad (24)$$

which has a typical ultraviolet behavior of the models for $\phi_\pm(\omega)$ adopted in [2] and obeys both the low-energy limit (22) and the normalization conditions (23). Calculation of the gluon correction in (20) with (24) then yields (at $\bar{\Lambda} = 0.6 \text{ GeV}$):

$$\int_0^\infty \frac{d\omega}{\omega} I(\omega) = \frac{\lambda_H^2 - \lambda_E^2}{9\bar{\Lambda}^3} \simeq 0.04 \text{ GeV}^{-1}. \quad (25)$$

We see that this correction is only at the level of 2% of our estimate for the leading order term in (20): $1/\lambda_B \sim 2 \text{ GeV}^{-1}$.

Hence, soft gluon effects are most probably suppressed both in the sum rule and in the WW relation. A future, more refined analysis of the B -meson 3-particle DA's using dedicated QCD sum rules (similar to one used in [8] for ψ_+) will allow to estimate these effects more accurately.

6 Conclusion

Summarizing, we have suggested a new approach to the B -meson DA, by relating it to the combination of two hadronic observables: the $B \rightarrow \pi$ form factor at zero momentum

transfer and f_B . The main result obtained in the zeroth order in α_s and neglecting soft gluon correction, is the sum rule for DA ϕ_-^B , which is converted into an equation for the inverse moment λ_B of ϕ_+^B using the WW relation between these two DA's. Our numerical estimate of λ_B agrees with the prediction of the two-point sum rule.

In order to avoid confusion, one has to emphasize that the procedure described above is completely different from deriving a factorization formula for $B \rightarrow \pi$ form factor in terms of B -meson DA. Such a formula exists [3] but it only involves the hard-scattering (factorizable) part of the form factor and the DA ϕ_+ . In our approach, the correlator calculated in terms of DA's is matched, via dispersion relation, with the full $B \rightarrow \pi$ form factor, or, in the absence of α_s -corrections, at least with the soft (nonfactorizable) part of that form factor.

We also presented qualitative estimates of soft-gluon corrections, assuming the asymptotic behaviour of 3-particle DA's. Our analysis indicates that the emission of the soft gluon contributing to the sum rule via 3-particle DA's, is $\sim 1/m_B$ suppressed, and that the WW relation is violated by a small correction. The radiative hard-gluon corrections to the sum rule, including renormalization effects, remain an important task, although the experience with the LCSR calculation of $B \rightarrow \pi$ form factor tells that at finite m_b , the $O(\alpha_s)$ effects are usually at a moderate level.

Acknowledgements

We are grateful to Th. Feldmann, M. Beneke and V. Braun for useful discussions and comments. We thank the authors of [23] for informing us about their results prior to publication. This work is supported by the German Ministry for Education and Research (BMBF).

References

- [1] A. Szczepaniak, E. M. Henley and S. J. Brodsky, Phys. Lett. B **243** (1990) 287.
- [2] A. G. Grozin and M. Neubert, Phys. Rev. D **55** (1997) 272.
- [3] M. Beneke and T. Feldmann, Nucl. Phys. B **592** (2001) 3.
- [4] G. P. Korchemsky, D. Pirjol and T. M. Yan, Phys. Rev. D **61** (2000) 114510;
S. Descotes-Genon and C. T. Sachrajda, Nucl. Phys. B **650** (2003) 356;
E. Lunghi, D. Pirjol and D. Wyler, Nucl. Phys. B **649** (2003) 349;
S. W. Bosch, R. J. Hill, B. O. Lange and M. Neubert, Phys. Rev. D **67** (2003) 094014.
- [5] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B **591** (2000) 313.
- [6] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B **612** (2001) 25;
S. W. Bosch and G. Buchalla, Nucl. Phys. B **621** (2002) 459;
A. Ali and A. Y. Parkhomenko, Eur. Phys. J. C **23** (2002) 89.

- [7] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385, 448 (1979).
- [8] V. M. Braun, D. Y. Ivanov and G. P. Korchemsky, Phys. Rev. D **69** (2004) 034014.
- [9] P. Ball and E. Kou, JHEP **0304** (2003) 029.
- [10] I. I. Balitsky, V. M. Braun and A. V. Kolesnichenko, Nucl. Phys. **B312** (1989) 509;
V. M. Braun and I. E. Halperin, Z. Phys. C **44** (1989) 157;
V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345** (1990) 137.
- [11] V. M. Belyaev, A. Khodjamirian and R. Ruckl, Z. Phys. C **60** (1993) 349;
V. M. Belyaev, V. M. Braun, A. Khodjamirian and R. Ruckl, Phys. Rev. D **51** (1995) 6177;
A. Khodjamirian, R. Ruckl, S. Weinzierl and O. I. Yakovlev, Phys. Lett. B **410** (1997) 275;
E. Bagan, P. Ball and V. M. Braun, Phys. Lett. B **417** (1998) 154;
P. Ball and R. Zwicky, JHEP **0110** (2001) 019.
- [12] B. O. Lange and M. Neubert, Phys. Rev. Lett. **91** (2003) 102001.
- [13] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B **358** (1995) 129;
A. Ali and V. M. Braun, Phys. Lett. B **359** (1995) 223.
- [14] H. Kawamura, J. Kodaira, C. F. Qiao and K. Tanaka, Phys. Lett. B **523**, 111 (2001) [Erratum-ibid. B **536**, 344 (2002)].
- [15] P. Colangelo and A. Khodjamirian, In: At the frontier of particle physics, vol. 3, 1495-1576 ed. by M. Shifman (World Scientific, Singapore, 2001). arXiv:hep-ph/0010175.
- [16] V. M. Braun and I. E. Halperin, Phys. Lett. B **328** (1994) 457;
V. M. Braun, A. Khodjamirian and M. Maul, Phys. Rev. D **61** (2000) 073004.
- [17] J. Bijnens and A. Khodjamirian, Eur. Phys. J. C **26** (2002) 67.
- [18] P. Ball, arXiv:hep-ph/0308249.
- [19] A. Khodjamirian, R. Ruckl, S. Weinzierl, C. W. Winhart and O. I. Yakovlev, Phys. Rev. D **62** (2000) 114002.
- [20] A. Khodjamirian, T. Mannel and M. Melcher, Phys. Rev. D **68** (2003) 114007.
- [21] P. Ball and R. Zwicky, Phys. Rev. D **71** (2005) 014015.
- [22] I. I. Balitsky and V. M. Braun, Nucl. Phys. B **311** (1989) 541.
- [23] F. De Fazio, T. Feldmann and T. Hurth, preprint SI-HEP-2005-03, arXiv:hep-ph/0504088.